

Trigonometrija - osnovne formule

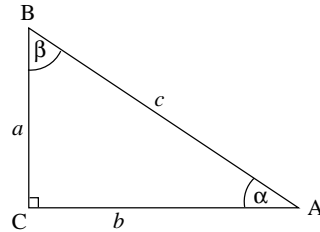
Definicija trigonometrijskih funkcija – u pravokutnom trokutu:

$$\sin \alpha = \frac{\text{nasuprotna kateta}}{\text{hipotenuza}} = \frac{a}{c}$$

$$\cos \alpha = \frac{\text{priležeća kateta}}{\text{hipotenuza}} = \frac{b}{c}$$

$$\operatorname{tg} \alpha = \frac{\text{nasuprotna kateta}}{\text{priležeća kateta}} = \frac{a}{b}$$

$$\operatorname{ctg} \alpha = \frac{\text{priležeća kateta}}{\text{nasuprotna kateta}} = \frac{b}{a}$$



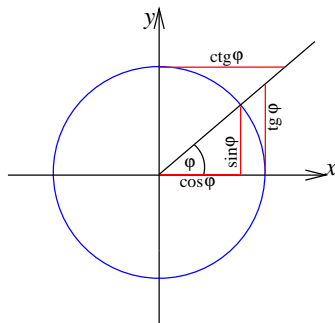
Vrijednosti trigonometrijskih funkcija u nekim važnijim kutevima:

stupnjevi:	0°	30°	45°	60°	90°	120°	150°	180°	270°	360°
radijani:	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}\pi$	$\frac{5}{6}\pi$	π	$1\frac{1}{2}\pi$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	0	$\pm\infty$	0
ctg	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$\mp\infty$	0	$\mp\infty$

Vrijednosti trigonometrijskih funkcija na jediničnoj kružnici:

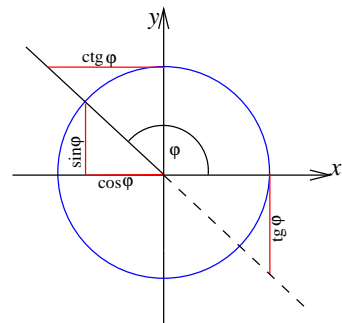
Prvi kvadrant:

$$\begin{aligned} \sin \varphi &> 0 \\ \cos \varphi &> 0 \\ \operatorname{tg} \varphi &> 0 \\ \operatorname{ctg} \varphi &> 0 \end{aligned}$$



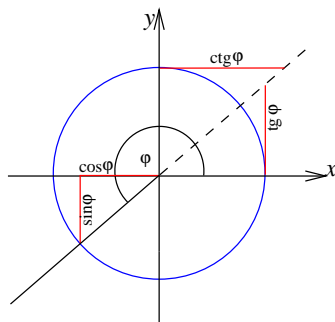
Drugi kvadrant:

$$\begin{aligned} \sin \varphi &> 0 \\ \cos \varphi &< 0 \\ \operatorname{tg} \varphi &< 0 \\ \operatorname{ctg} \varphi &< 0 \end{aligned}$$



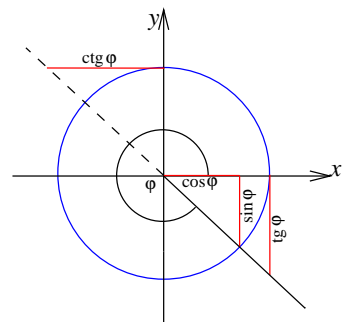
Treći kvadrant:

$$\begin{aligned} \sin \varphi &< 0 \\ \cos \varphi &< 0 \\ \operatorname{tg} \varphi &> 0 \\ \operatorname{ctg} \varphi &> 0 \end{aligned}$$



Četvrti kvadrant:

$$\begin{aligned} \sin \varphi &< 0 \\ \cos \varphi &> 0 \\ \operatorname{tg} \varphi &< 0 \\ \operatorname{ctg} \varphi &< 0 \end{aligned}$$



Osnovne trigonometrijske formule:

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$$

• **Parnost i neparnost trigonometrijskih funkcija:**

$$\sin(-\alpha) = -\sin \alpha$$

$$\cos(-\alpha) = \cos \alpha$$

$$\operatorname{tg}(-\alpha) = -\operatorname{tg} \alpha$$

$$\operatorname{ctg}(-\alpha) = -\operatorname{ctg} \alpha$$

• **Izražavanje jedne trigonometrijske funkcije pomoću druge:**

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \frac{\operatorname{tg} \alpha}{\sqrt{1 + \operatorname{tg}^2 \alpha}} = \frac{1}{\sqrt{1 + \operatorname{ctg}^2 \alpha}}$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \frac{1}{\sqrt{1 + \operatorname{tg}^2 \alpha}} = \frac{\operatorname{ctg} \alpha}{\sqrt{1 + \operatorname{ctg}^2 \alpha}}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\sqrt{1 - \sin^2 \alpha}} = \frac{\sqrt{1 - \cos^2 \alpha}}{\cos \alpha} = \frac{1}{\operatorname{ctg} \alpha}$$

$$\operatorname{ctg} \alpha = \frac{\sqrt{1 - \sin^2 \alpha}}{\sin \alpha} = \frac{\cos \alpha}{\sqrt{1 - \cos^2 \alpha}} = \frac{1}{\operatorname{tg} \alpha}$$

• **Funkcije zbroja i razlike kutova:**

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\operatorname{tg}(\alpha \pm \beta) = \frac{\operatorname{tg} \alpha \pm \operatorname{tg} \beta}{1 \mp \operatorname{tg} \alpha \cdot \operatorname{tg} \beta}$$

$$\operatorname{ctg}(\alpha \pm \beta) = \frac{\operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta \mp 1}{\operatorname{ctg} \beta \pm \operatorname{ctg} \alpha}$$

• **Funkcije dvostrukih kutova:**

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2 \alpha - 1}{2 \operatorname{ctg} \alpha}$$

• **Funkcije polovičnog kuta:**

$$\sin \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\operatorname{tg} \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha}$$

$$\operatorname{ctg} \frac{\alpha}{2} = \sqrt{\frac{1 + \cos \alpha}{1 - \cos \alpha}} = \frac{1 + \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 - \cos \alpha}$$

• **Zbroj i razlika trigonometrijskih funkcija:**

$$\sin \alpha + \sin \beta = 2 \cdot \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cdot \cos \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cdot \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \cdot \sin \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}$$

$$\operatorname{tg} \alpha \pm \operatorname{tg} \beta = \frac{\sin(\alpha \pm \beta)}{\cos \alpha \cdot \cos \beta}$$

$$\operatorname{ctg} \alpha \pm \operatorname{ctg} \beta = \pm \frac{\sin(\alpha \pm \beta)}{\sin \alpha \cdot \sin \beta}$$

$$\operatorname{tg} \alpha + \operatorname{ctg} \beta = \pm \frac{\cos(\alpha - \beta)}{\cos \alpha \cdot \sin \beta}$$

$$\operatorname{ctg} \alpha - \operatorname{tg} \beta = \pm \frac{\cos(\alpha \pm \beta)}{\sin \alpha \cdot \cos \beta}$$

• **Produkt funkcija:**

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

• **Rješavanje trokuta:**

• Sinusov poučak: $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$ ili $a : b : c = \sin \alpha : \sin \beta : \sin \gamma$

• Kosinusov poučak: $a^2 = b^2 + c^2 - 2bc \cdot \cos \alpha$ $b^2 = a^2 + c^2 - 2ac \cdot \cos \beta$ $c^2 = a^2 + b^2 - 2ab \cdot \cos \gamma$
 $\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos \beta = \frac{a^2 + c^2 - b^2}{2ac}$ $\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab}$

• **Jednostavne trigonometrijske jednadžbe:**

$\sin x = a$	$x = (-1)^k \sin^{-1} a + k\pi$
$\cos x = a$	$x = \pm \cos^{-1} a + 2k\pi$
$\operatorname{tg} x = a$	$x = \operatorname{tg}^{-1} a + k\pi$
$\operatorname{ctg} x = a$	$x = \operatorname{ctg}^{-1} a + k\pi$